Listing of the

Rockefeller Mathematics Seminar
Spring 1997

Abstracts by the speakers.

Mathematical Crystals and Quasi-Crystals

J. C. Lagarias, AT& T, New Jersey.

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February 6.

In 1984 Schechtman et al discovered materials whose X-ray diffraction spectrum showed sharp spots indicating long-range order, but with icosahedral symmetry which is impossible for a periodic lattice. Such matrials must therefore have a nonperiodic arrangement of atoms. There has been intense work in the last ten years by physicists and mathematicians concerning possible structure of such materials. There is a strong number theoretic flavor to such structures. This talk will survey various results on mathematical models for crystals and quasicrystals in n dimensions.

Newton's Method

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February 13.

A method is described to numerically find roots of a polynomial of arbitrary degree d. Suppose the roots are all contained in the unit disk. Then the method consists of applying Newtons method to roughly $d^{3/2}$ points that are equally spaced on the circle of radius 2 or more centered at the origin. This method can be proved to yield all the zeros of the polynomial. Other methods, for example with variable stepsize are discussed, but in general their convergence has not been proved. Examples are given and the connection with Julia sets is discussed.

Turning Surfaces Inside Out

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February 27.

The problem is how to turning a surface inside-out, keeping it "immersed," i.e. a non-singular surface, possibly with self-intersections, at each stage. One solution will be described, and another solution, due to William Thurston, will be shown on video. The problem fits into a theory of cobordism classes of immersed surfaces in 3-space; these form a group of order 8 which will also be illustrated by a video.

Hyperbolicity and Renormalization

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March 6.

Renormalization has been used as a tool to understand the dynamics of unimodal maps starting with the work of Feigenbaum some twenty ago. Crucial in this undertsnding is the fact that in the appropiate function space renormalization has an invariant set on which it acts hyperbolically. The setting is a space of degree two polynomial-like maps. We prove this conjecture in the space of quadratic-like maps for all real combinatorial types.

Hamiltonian Round-Off

J. H. Lowenstein, NYU, and F. Vivaldi, QMW, London

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March 20.

We study the propagation of round-off errors near the periodic orbits of an area-preserving map, discretized on a lattice in such a way as to retain reversibility. We show that all orbits are periodic, and that they form a quasi-crystallic structure with asymptotic scaling properties, characterized by a fractal dimension. We calculate the round-off error probability distribution as a function of time t , and prove that its moments increase asymptotically as positive powers of t , modulated by functions which are periodic in the logarithm of t . Thus, the diffusion coefficient fails to exist, and the diffusion is anomalous.